INFLATABLE MEMBRANE MIRRORS FOR OPTICAL PASSBAND IMAGERY

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Abstract Inflatable membrane mirrors are currently technically not feasible for use in optical passbands. We will review the status of this technology, its problems and possible new technologies that may enable their use. We will present results concerning membranes having a radial thickness gradient.

Subject terms : space optics, inflatable membrane mirrors, visible passband inflatable mirrors

1. Introduction

The emphasis for small, fast and cheap spacecraft makes it highly desirable to find the means for packaging large optical systems into a small lightweight package for launch. The optical system then is unfolded or otherwise deployed after insertion into orbit. An inflatable membrane mirror is clearly the ultimate lightweight mirror. Past investigations, however, have led to the conclusion that they are not suitable for imaging in the visible or optical infrared. Current studies have shown that inflatable structures, rigidizable in space, offer the means for compact packaging, deploying into very large structures of sufficient rigidity to serve as the optical truss subsystem of large telescopes or cameras. The integration of inflatable optics into such a package is clearly a desirable goal. Our current study has re-examined the problems that have been identified by others and we discuss today some that can be solved. Other problems remain, limiting applications to the infrared. However, new materials and techniques are on the horizon that may make inflatable membrane mirrors applicable to the CCD passband.

Inflated membrane mirrors consist of two thin circular membranes that are sealed on their periphery, attached to a tensioning ring and inflated to a pressure sufficient to produce the focal ratio required of the mirror. Such inflated mirrors have been used for various applications wherein low imaging acuity is sufficient; for instance, microwave antennae[1,2,3] and solar energy concentrators[4, 5]. They were never contemplated for optical passband imaging where high acuity is required. The basic problem is that the shape of an inflated membrane is neither spherical nor parabolic. The profile of such a mirror has an up-turned periphery with regard to a sphere and hence is an oblate spheroid. Its shape is generally expressed by an even power series termed the Hencky curve, after the first person to explore such mirrors[6]. The resulting spherical aberration is serious and forces placing a secondary mirror for a Cassegrain configuration far from the prime focus, as shown in Fig. 1. Hence an unconventionally large secondary mirror is required

With renewed interest in the possibility of using inflation-deployed optics in space systems it is appropriate to re-examine the issues surrounding the use of inflatable optics. The challenges facing application of inflatable membrane mirrors include:

1. Solving the Hencky shape of the inflated mirror surface.
2. Non-uniformity of membrane mechanical properties and of the inflated surface.
3. Non-uniformity of optical properties of transparent window membranes.
4. IR transparency of membrane windows.
5 Obtaining large monolithic membrane sheets (gores are a serious problem).
6. Creases and other handling and packaging operations.
7. Maintenance of pressure (focus, optical figure, collimation, etc.).

Today we will discuss attainment of an optically useful inflated shape of the membrane.

2. Membrane shape

We have re-examined issues relating to the shape of an inflated membrane and the conclusion that they are not feasible for optical passband applications. One problem that has been discussed in various reports cited above is the intrinsic shape of the inflated surface and asymmetry in the inflated shape. The shape of a uniformly thick membrane is generally described as the "Hencky curve." It is an oblate spheroid which produces a caustic that cannot be integrated into a conventional Cassegrain configuration.

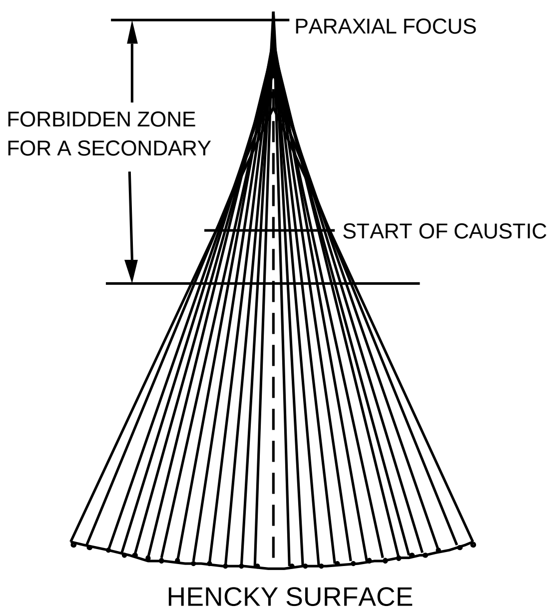

Fig. 1 The caustic formed by the membrane mirror analyzed in this paper. A secondary mirror must be located considerably below the start of the caustic, resulting in a large central obscuration and severe coma.

The challenge to transform the Hencky curve into a paraboloid can be better appreciated by comparing it to a paraboloid. For convenience of comparison one can write the equations for a Hencky surface, a spherical surface and a paraboloid in similar form as:

$z(u) = (D/64F^2)(u^2 + 0.1111\, u^4)$	Hencky	(1 a)
$z(u) = (D/64F^2)(u^2 + 0.0156\, u^4)$	Sphere	(1 b)
$z(u) = (D/64F^2)(u^2 + 0.0000\, u^4)$	Paraboloid.	(1 c)

This shows that going from the Hencky curve to a sphere is much larger than going from a sphere to a paraboloid.

When a paraboloid is subtracted from measurements of inflated membranes the resulting curve has been termed a "W-curve." Published W-curves[7] usually have a large asymmetry that has been interpreted as indicating an asymmetry in the inflated shape of the membrane. Because this asymmetry has been accepted as being real it is important to re-analyze the original measurements and apply the appropriate corrections for the metrology zero point.

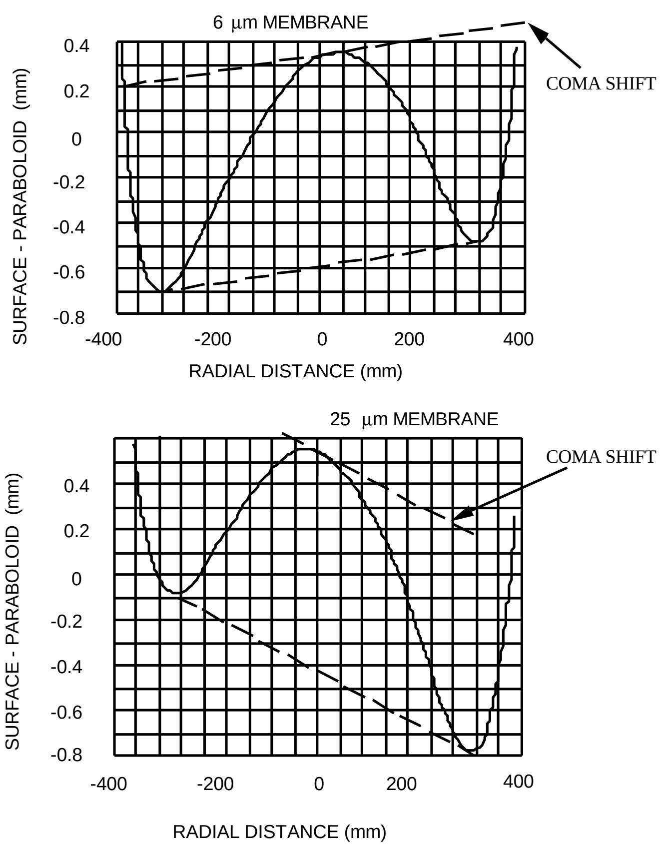

Fig. 2 Two examples of measurements of inflated membrane mirrors showing an asymmetrical W-curve.

3. Re-analysis of the l'Garde data

Cassapakis[8] of l'Garde has kindly provided us with the measurement data for a 900 mm diameter, 6 um thick Mylar membrane mirror having a central radius of curvature of 2070 mm. The customary way to present this data in graphical form is to subtract a best fit parabolic curve, which results in the "W-curve." An example of this asymmetry is shown in Fig. 2. We have added a straight, but tilted, line tangent to the minima of these W-curves. This tilt is familiar to opticists as being an artifact caused by a small error in the zero-point of the metrology system, producing an error generally termed "coma." The data provided to us did not show this large asymmetry, as shown in Fig. 3, and was corrected by a small shift in the r and z coordinates of the data. The sagitta vs. radial zone is shown at the top and the difference with respect to a paraboloid at the bottom.

The very small residuals shown in Fig. 3 indicate that this membrane was symmetrical, which raises the issue of the curve data for an azimuth in the transverse direction. In Fig. 4 we show the measured membrane sagitta minus the same parabolic curve for the cuts in orthogonal directions. This difference in slope means that the oblate ellipsoid is astigmatic

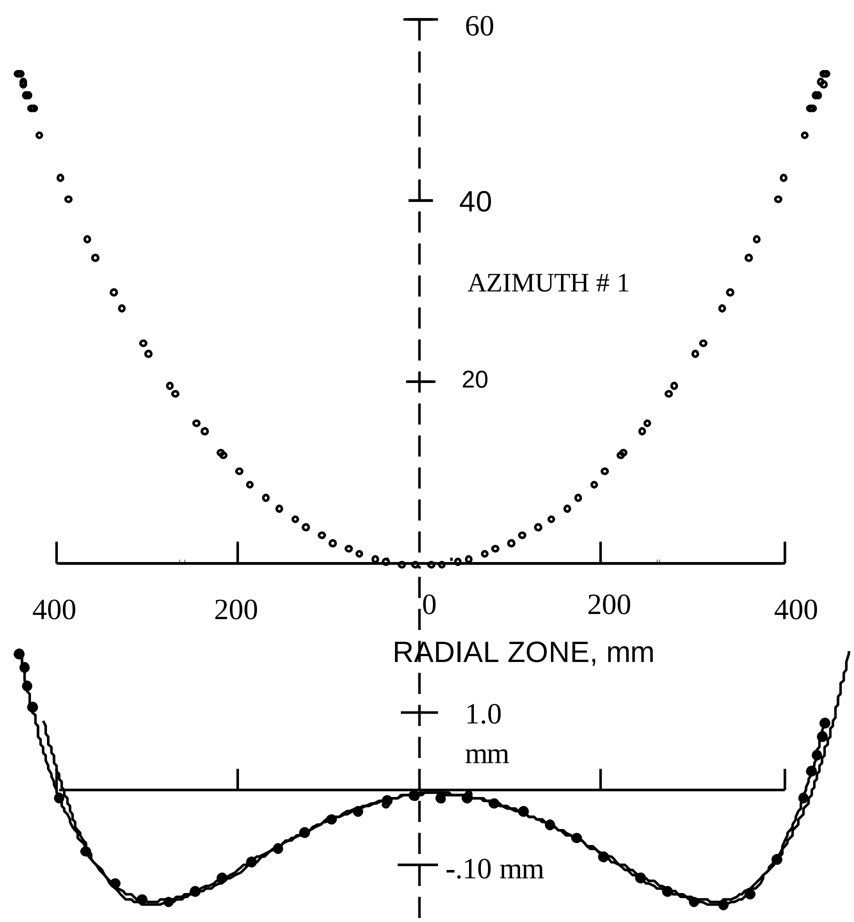

Fig. 3 Data for the 6 um Mylar membrane mirror, the sagitta (top) and the difference with respect to a paraboloid (bottom).

and to a degree that would produce a serious image error, assuming that the system could otherwise handle this oblate shape. Measurements at l'Garde[9] found that Mylar is indeed orthotropic, the modulus in orthogonal directions differing by 44 %, but that it varies from batch to batch. They find that Kapton E is very close to isotropic.

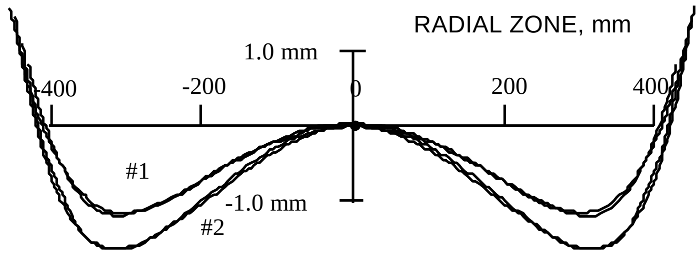

Fig. 4 Comparison of the residuals after subtraction of the same parabolic curve from both azimuths shows that there is a significant difference in the curves in these orthogonal directions.

This astigmatic shape of the membrane is quite different from the astigmatism encountered in optical designs. It switches axes between the outer and inner regions as shown in Fig. 5. This effect is because the low E direction is flatter near the center and steeper near the periphery. This leads to a form of astigmatism different from that encountered in conventional optics. Fig. 5 (right) shows the contours for equal slopes where the axis switches 90° from the central to the peripheral regions. Correction for this type of astigmatism can only be done at a wavefront corrector placed at a pupil.

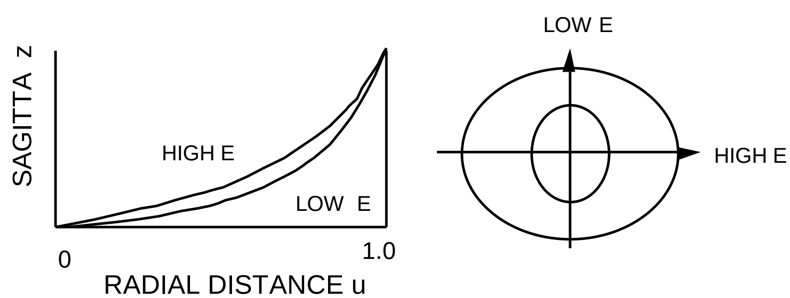

Fig. 5. Schematic representation of the type of astigmatism caused by a variation of the membrane value of E (or thickness t) in orthogonal directions showing a 90° change in the major axis between the inner and outer regions of the mirror surface.

4. Variable thickness membrane mirrors

Use of a membrane having a radial variation of thickness is one option for solving the Hencky shape problem. The equation for the central deflection, z, of a circular thin plate to that for a thin membrane can be written as the equation for the inverse central deflection as the sum of two standard terms:

$$1/z = \underbrace{Et^3 / k_1 P r^3}_{\text{bending mode}} + \underbrace{E^{1/3} t^{1/3} / k_2 P^{1/3} r^{4/3}}_{\text{stretching mode}} \qquad (3)$$

Thus for a thin membrane we have

$$z = 0.248P^{1/3}D^{4/3} [0.9au^2 + 0.1u^4 + ...] \div E^{1/3}t^{1/3} \quad = 1 \tag{4}$$

with u larger than 0 but less than 1. Now let the variation of thickness with x be $t = f(t,u) = t\{1 + Au^2 + Bu^4 + ...)$ then

$$z = 0.248P^{1/3}D^{4/3} (0.9u^2 + 0.1u^4) \div E^{1/3}f(t,u)^{1/3}. \tag{5}$$

The change in z as a function of $u = 2x/D$ and $du = (2/D)dx$, is then

$$dz/du = (k /E^{1/3}t^{1/3})\{[(1.8u + 0.4u^3) \div (1 + Au^2)^{1/3}] - [2 Au (0.9u^2 + 0.1u^4) \div 3(1+ Au^2)^{4/3}]\}. \tag{6}$$

Note that A is independent of P, E, t and central sag z of the inflated membrane. We solve for the value of A such that the u^3 term in Eq. 14 is zero at u = 1. Collecting the u^3 terms and solving for A, which means a paraboloid, we have
$A = 4(0.1) \div 0.9 = 0.44$.

One can select A not exactly equal to 0.44 so that the resulting membrane has a small u^4 term as would be required if the membrane were to serve as a convex hyperbolic Cass secondary. Likewise it could be selected so as to produce the aspheric shape of a Ritchey-Chretien primary mirror.

Returning to Eq. (6), the sagitta of the inflated membrane, z, is then the integral of Eq. (6). Since there is no closed form for this integral we have to evaluate it by numerical integration.

$$z = \Sigma (dz/du_i + dz/du_j)\Delta/2. \tag{7}$$

The optimum value of A was determined by iterations of Eq (7) to be 0.42, which is close to the theoretical value of 0.44.

A plot of the radial variation in membrane slope for a membrane of constant thickness is shown in Fig. 6, showing the up-turned outer regions of the Hencky curve (open circles). The slope for the membrane having a parabolic radial gradient is shown by the small circles, the straight line being the constant slope of a paraboloid.

The residual between the variable thickness membrane and a linear (parabolic) curve is shown in Fig. 7. This indicates that higher order terms, u^4 and u^6, will be necessary in order to get the surface slope residuals down to the order of 2.5e-06 rad, or a ray deviation of 1 arc second. This, in turn, emphasizes the accuracy with which the variation of thickness must be done. The edge thickness was increased by 2.52 um for the residuals shown in Fig. 7. To further reduce the slope error from ±0.0005 rad down to 2.5e-06 rad requires an improvement factor of 200.

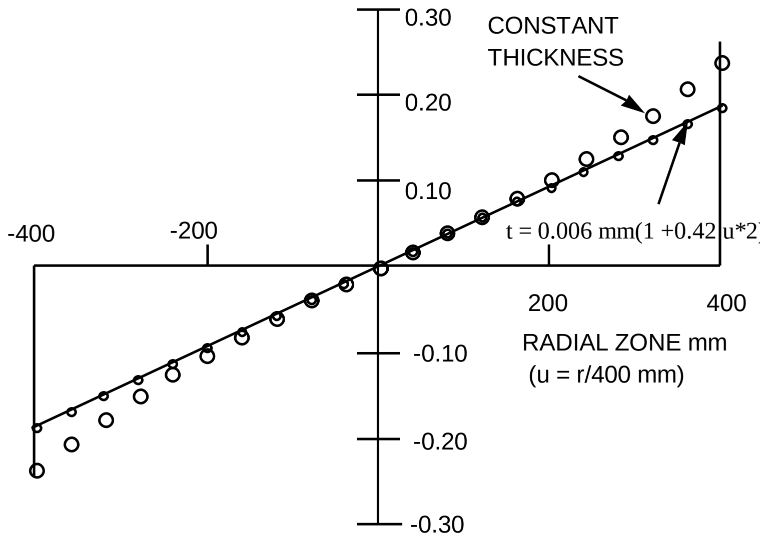

Fig. 6 Variation of the membrane slopes vs. zonal radius, r, for a uniformly thick membrane (large circles) and a variable thickness membrane (small circles) with a linear (parabolic) relationship shows the ability to control the surface figure by increasing the membrane thickness with increasing radial distance.

6 Fabrication of a variable thickness membrane

6.1 *Flat membrane*

Fabrication is a key step in forming a membrane having a monotonically varying radial thickness gradient. Forming a uniform film by pouring a polymer onto a rotating flat plate

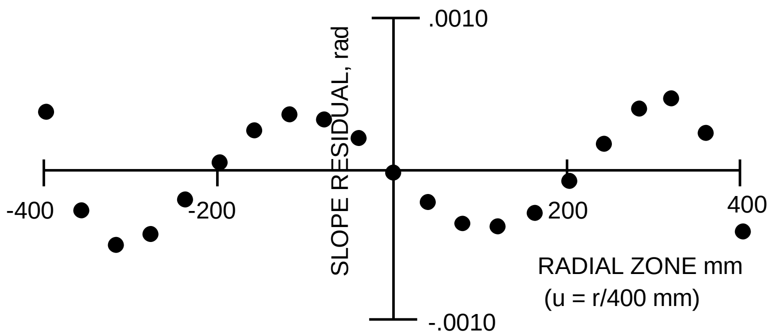

Fig. 7 The residual between the variable thickness membrane and the straight line relationship of Fig. 11 showing that a moderate dependence on thickness on u^4 and u^6 is present.

has been done. One can modify the process to form a non-uniform membrane. We have shown above that the ideal radial variation in membrane thickness is parabolic. The process for manufacturing such a parabolic thickness membrane to the desired high precision is to use a rotating table or mold.

There are two options:

1. A slowly rotating flat plate mold having a dike at the periphery to allow the liquid to accumulate at the periphery or

2. A fixed plate having a very long radius convex spherical curvature.

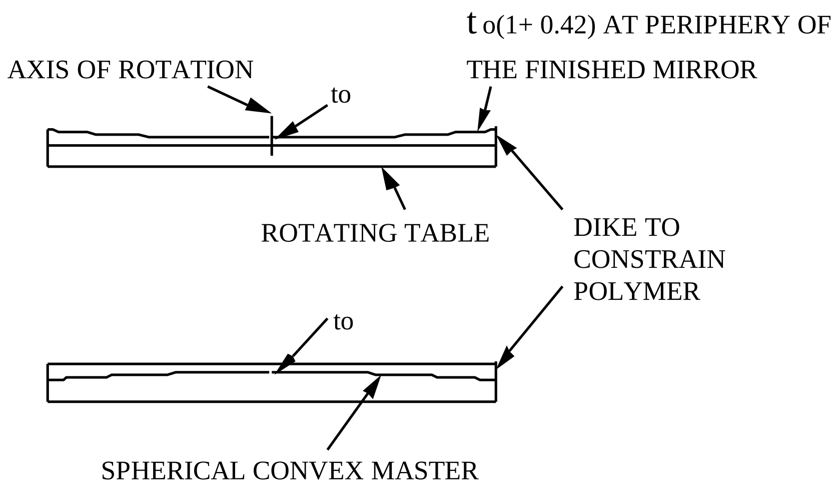

Fig. 8 Two fabrication modes 1) A flat table carrying a flat sheet of glass is spun so that the edge thickness of the liquid polymer is that required to produce the edge thickness as illustrated above, and 2) a convex spherical polished master on which the liquid polymer is poured.

Fig. 8, top, shows two options for forming a flat membrane having a parabolic radial thickness variation required to yield a paraboloid upon inflation. The first option (top) is to use a rotating table. A circular flat sheet of glass or other suitable material can be used for this table. A dike is placed at the periphery of the mold to contain the liquid polymer. The table is then rotated at an angular velocity such that the edge thickness at the planned diameter of the membrane (at u = 1.0) is 1.42 times the central thickness of the spun liquid as specified by Eqs (25-27). The polymer is then polymerized while the table continues to spin.

For a 1-meter diameter table and $t_0 = 0.01$ cm and t = 0.0143 at u = 1, T = 59.3 sec/rev. If one were to make a 20 meter diameter parabolic reflector for far infrared, for example, the rotation period for a membrane $t_0 = 0.10$ cm the rotation period would be T = 377 sec/rev.

The second option (Fig. 8, bottom) is to use a fixed table and a slightly convex spherical master. The spherical curve produces the quadratic radial variation in thickness. This option would be preferred when one wants to make a number of identical flat membranes. For a 100 um central thickness membrane and a 1 meter diameter substrate the radius of curvature would be 880 meters.

In both cases the surface of the mold should be polished so that the optical surface of the finished membrane replicates the polished surface of the master. The free surface of the polymerized polymer on the other hand can have small surface irregularities that would affect the specularity of the finished membrane.

6.2 *Curved membrane*

A pre-curved membrane may be useful inasmuch as the inflation pressure can be reduced over that required with a flat membrane. In Fig. 9 we illustrate the formation of a parabolic shaped membrane having a parabolic radial thickness gradient.

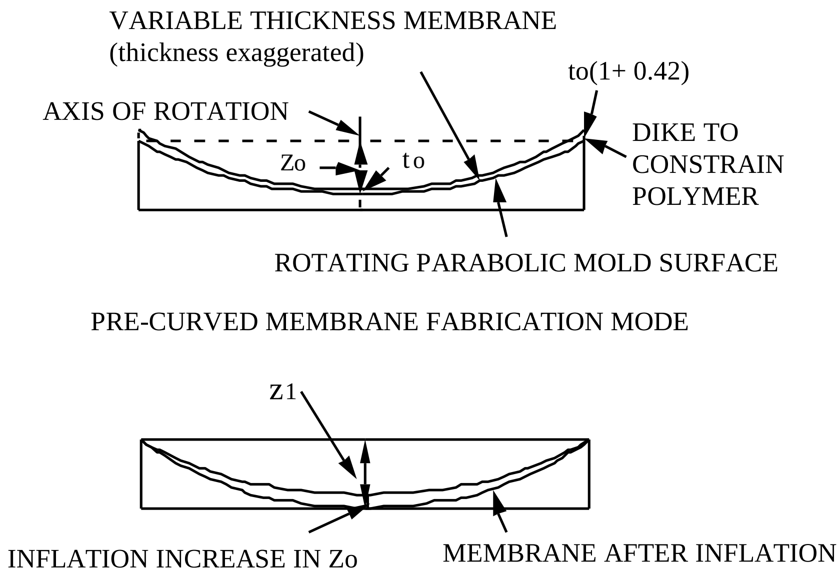

Fig. 9 (Top) The sequence shows the fabrication mode for forming a membrane having a parabolic radial variation in thickness. (Bottom) The added curve depth after inflation produces the required focal ratio of the mirror.

The steps are as follows:

1) Spin cast a substrate to form the parabolic mold upon which the membrane will be formed.

2) Place a dike around the periphery to contain the spun liquid polymer.

3) The polymer is then poured into the pre-fabricated mold paraboloid and spun at the slightly different angular velocity as specified by Eq. (29), such that the edge thickness of the membrane is 1.42 times the central thickness at the periphery of the finished mirror, namely at $u = 1.00$. Note that $u = 1$ is not necessarily the same as the outer diameter of the inflated mirror for the reasons we have already stated.

4) Remove the thin membrane, ready for use. Here the problem is which parting medium to use so that the membrane can be easily separated from the glass (or suitable polymer parabolic mold substrate).

For an example of a 100 cm diameter F/1.1 membrane of central thickness 0.01 cm we have for the mold paraboloid

$$T = (\pi^2\ 110\ \text{cm} \div 980\ \text{cm/sec}^2)^{1/2} = 2.9770\ \text{sec/ rev.} \qquad (8)$$

To form a parabolic liquid surface of additional edge thickness of 0.0143 cm on this mold we need to change the angular velocity by ΔT seconds.

$$\Delta T = 0.0037\ \text{second, thus}\ (T + \Delta T) = 2.9807\ \text{sec/rev.} \qquad (9)$$

7 Practical aspects

One of several configurations of an inflatable optical system spacecraft that we have explored[8] is shown in Fig. 10. The optical cavity is formed by a cylinder of inflated rings which

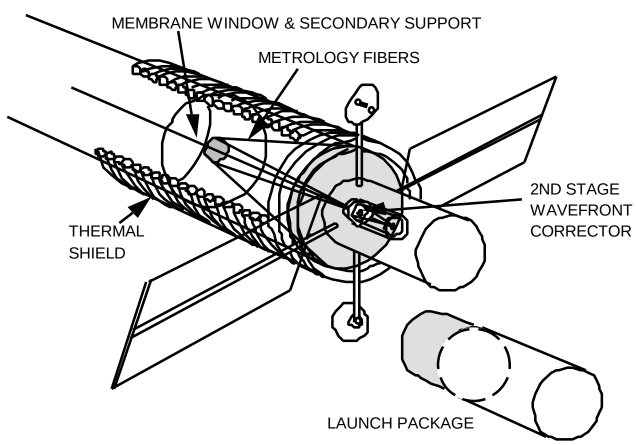

Fig. 9. Schematic cross-section of an inflatable space telescope using inflated rings plus metrology fibers to locate the position of the secondary mirror, showing the compactness that is possible using an inflated membrane mirror & associated structure.

position the secondary mirror and also provide a thermal shield. The planes of both the primary mirror and of the small secondary mirror are defined by the interface between two adjacent inflated rings The carbon fiber metrology truss between the primary mirror and the secondary mirror to define and maintain focus andcollimation is also shown.

8 Conclusions

The potential for a compact launch package shows the reason why continuation of work in inflatable optical systems is justified, the practical problems not withstanding. A number of conclusions can be drawn from this study.

1) Commercial Mylar has enough variation of physical properties with azimuth that an inflated 6 um thick Mylar membrane mirror will have astigmatic behavior of such a magnitude that it is doubtful if it can be used in a high-acuity optical passband system. Other polymers, like Kapton, may alleviate this problem.

2) A parabolic variation of the radial thickness of a membrane will become a close approximation to a paraboloid upon inflation. A value of A = 0.42 is found for the au^2 thickness profile term independent of thickness or inflation pressure. Further tune-up of the wavefront, however, is required to get within 1 asec.

3) The required radial parabolic variation in membrane thickness can be produced either by spin casting on a flat surface or spin casting on a pre-formed parabolic mold.

In summary, there are a host of other issues that must be resolved before inflatable membrane mirrors can be seriously considered for space optical systems. The potential advantage of low-cost, large-aperture space telescopes for use in the 0.5 to 12 um region hangs in the balance.

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